Worksheet 9 Numerical Analysis Spring 2023

Name: ____________________ NetID: | | | | | | | | | | | |

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Work in groups of at least 2 and at most 4.

Consider the two algorithms which take in a matrix **A** and a vector **b**, and output an approximation to the largest magnitude eigenvector:

$\mathbf{y} \leftarrow \mathbf{b}$
for $i = 1, 2, \ldots, k$:
    $\mathbf{y} \leftarrow \mathbf{A}\mathbf{y}$
$\mathbf{y} \leftarrow \mathbf{y}/\|\mathbf{y}\|$ return $\mathbf{y}$

$\mathbf{y} \leftarrow \mathbf{b}$
for $i = 1, 2, \ldots, k$:
    $\mathbf{y} \leftarrow \mathbf{A}\mathbf{y}$
    $\mathbf{y} \leftarrow \mathbf{y}/\|\mathbf{y}\|$
return $\mathbf{y}$

How do the outputs compare?

Suppose $\mathbf{A}$ has eigenvalue decomposition:

$$\mathbf{A} = \mathbf{V}\begin{bmatrix} -4 & & & \\ & -1 & & \\ & & 2 & \\ & & & 3 \end{bmatrix}\mathbf{V}^{-1}, \qquad \mathbf{V} = \begin{bmatrix} | & | & | & | \\ \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 & \mathbf{v}_4 \\ | & | & | & | \end{bmatrix}, \qquad \mathbf{x} = \mathbf{V}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

1. Write the eigendecomposition of $\mathbf{A} + c\mathbf{I}$

2. Write the eigendecomposition of $(\mathbf{A} + c\mathbf{I})^{-1}$

3. Suppose $c = 2.2$. What are the eigenvalues of $(\mathbf{A} + c\mathbf{I})^{-1}$?

4. For this value of c, what happens if we apply power method with $(\mathbf{A} + c\mathbf{I})^{-1}$?