Worksheet 8 — Numerical Analysis Spring 2023

Name: ____________________ NetID: | | | | | | | | | | | |

Name: ____________________ NetID: | | | | | | | | | | | |

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Work in groups of at least 2 and at most 4.

Suppose $\mathbf{A}$ has eigenvalue decomposition:

$$\mathbf{A} = \mathbf{V}\begin{bmatrix} -4 & & & \\ & -1 & & \\ & & 2 & \\ & & & 3 \end{bmatrix}\mathbf{V}^{-1}, \qquad \mathbf{V} = \begin{bmatrix} | & | & | & | \\ \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 & \mathbf{v}_4 \\ | & | & | & | \end{bmatrix}, \qquad \mathbf{x} = \mathbf{V}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

What is $\mathbf{V}^{-1}\mathbf{v}_2$?

Show that $\mathbf{v}_3$ is an eigenvector of $\mathbf{A}$, what is the corresponding eigenvalue?

$$\mathbf{A} = \mathbf{V}\begin{bmatrix} -4 & & & \\ & -1 & & \\ & & 2 & \\ & & & 3 \end{bmatrix}\mathbf{V}^{-1}, \qquad \mathbf{V} = \begin{bmatrix} | & | & | & | \\ \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 & \mathbf{v}_4 \\ | & | & | & | \end{bmatrix}, \qquad \mathbf{x} = \mathbf{V}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

What is $\mathbf{Ax}$? What about $\mathbf{A}^2\mathbf{x}$? What about $\mathbf{A}^k\mathbf{x}$.

How can we use this to obtain a value near to an eigenvector of $\mathbf{A}$ given we know $\mathbf{A}$ and $\mathbf{x}$, but not their factorizations in terms of $\mathbf{V}$?

What if $\mathbf{x} = \mathbf{V}[x_1\ x_2\ x_3\ x_4]^\top$ instead of $\mathbf{V}[1\ 1\ 1\ 1]^\top$?