Worksheet 5 Numerical Analysis Spring 2023

Name: _____ NetID: | | | | | | | | | | |

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Work in groups of at least 2 and at most 4.

Problem 1. Suppose

$$\mathbf{U} = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}, \quad \mathbf{\Sigma} = 2\sqrt{2}\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{V}^\mathsf{T} = \frac{1}{\sqrt{2}}\begin{bmatrix} \sqrt{2} & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

What is $\mathbf{U\Sigma V}^\mathsf{T}$?

For $i = 1, 2, 3$, let $\mathbf{u}_i$ be the i-th column of $\mathbf{U}$ and $\mathbf{v}_i^\top$ be the i-th row of $\mathbf{v}$. Compute $\mathbf{u}_i \mathbf{v}_i^\top$.

Compute $\sum_{i=1}^{3} \sigma_i \mathbf{u}_i \mathbf{v}_i^\top$, where σ_i are the diagonal entries of $\mathbf{\Sigma}$.

Use this to find a 4×3 matrix $\tilde{\mathbf{U}}$, a 3×3 diagonal matrix $\tilde{\mathbf{\Sigma}}$ and a 3×3 matrix $\tilde{\mathbf{V}}^\top$ such that $\mathbf{U}\mathbf{\Sigma}\mathbf{V}^\top = \tilde{\mathbf{U}}\tilde{\mathbf{\Sigma}}\tilde{\mathbf{V}}^\top$