Worksheet 4 Numerical Analysis Spring 2023

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Work in groups of at least 2 and at most 4.

Problem 1. Suppose

$$\mathbf{A} = \begin{bmatrix} 10^{-20} & 1 \\ 1 & 1 \end{bmatrix}.$$

1. Find a matrix $\mathbf{L}^{(1)}$ which introduces a zero in the bottom left corner by subtracting some multiple of the first row from the second row.

2. Find the corresponding LU factorization for $\mathbf{A}$.

3. One of the entries of your LU factorization should be $1 - 10^{20}$. On a computer, this might get rounded to -10^{20}. Let $\tilde{\mathbf{L}}$ and $\tilde{\mathbf{U}}$ be the factors after this rounding error. How do the factors compare to $\mathbf{L}$ and $\mathbf{U}$? How does $\tilde{\mathbf{L}}\tilde{\mathbf{U}}$ compare to $\mathbf{A}$?

Problem 2. Suppose

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & -2 \\ -7 & 0 & 3 \\ 1 & 3 & 4 \\ 3 & 1 & 2 \end{bmatrix}, \quad \text{or} \quad \mathbf{A} = \begin{bmatrix} 2 & 4 & -2 \\ 0 & -3 & 2 \\ 1 & 1 & 3 \\ 2 & -2 & 1 \end{bmatrix}$$

1. Find the matrix which extracts the i-th column of $\mathbf{A}$. I.e. which returns a vector containing just the i-th column. Should you multiply this matrix from the left or from the right?

2. Find the matrix with places a column vector into a matrix of all zeros in the j-th column. Should you multiply this matrix from the left or right?

3. Put these together to find a matrix with places the i-th column of $\mathbf{A}$ into the j-th column of a matrix of zeros.

4. Find a matrix which permutes the columns of $\mathbf{A}$ from $(1, 2, 3) \rightarrow (3, 1, 2)$.

5. Apply this matrix to permute the columns of the identity matrix. What does this tell you about how you could construct this matrix?