Worksheet 3 Numerical Analysis Spring 2023

Name: ______________________ NetID: | | | | | | | | | | |

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Work in groups of at least 2 and at most 4.

Problem 1. Write down code to solve a general upper triangular linear system

$$\begin{bmatrix} u_{1,1} & u_{1,2} & \cdots & u_{1,n} \\ & u_{2,2} & \cdots & u_{2,n} \\ & & \ddots & \vdots \\ & & & u_{n,n} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

How many floating point operations are required by your algorithm?

Problem 2. Suppose for $n \times n$ matrices $\mathbf{A}, \mathbf{B}$ we compute $\mathbf{C} = \mathbf{A}\mathbf{B}$ by

$$[\mathbf{C}]_{i,j} = \sum_{k=1}^{n} [\mathbf{A}]_{i,k} [\mathbf{B}]_{k,j}, \qquad i, j = 1, 2, \ldots, n.$$

How many floating point operations does this take? For what ω is this $O(n^{\omega})$?

Problem 3. Show $10n \log(\log(n)) = O(n \log(n))$. How big does n need to be so that $10n \log(\log(n)) < n \log(n)$?

Problem 4. Define,

$$\mathbf{A} = \begin{bmatrix} 3 & 4 & 1 & -8 \\ 6 & 5 & 2 & -28 \\ 3 & 2 & -8 & -13 \\ -12 & -15 & -4 & 48 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} -10 \\ -26 \\ -38 \\ 54 \end{bmatrix}$$

We want to find $\mathbf{x}$ so that $\mathbf{Ax} = \mathbf{b}$. Suppose we have

$$\mathbf{L} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & -3 & 0 & 0 \\ 1 & -2 & 3 & 0 \\ -4 & 1 & 0 & 4 \end{bmatrix}, \qquad \mathbf{U} = \begin{bmatrix} 3 & 4 & 1 & -8 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & -3 & 1 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

Verify $\mathbf{A} = \mathbf{LU}$

Use this fact to solve $\mathbf{Ax} = \mathbf{b}$.

Problem 5. Define,

$$\mathbf{A} = \begin{bmatrix} 1 & 5 & 7 & 8 \\ 1 & -1 & 1 & 6 \\ 1 & 5 & 5 & -2 \\ 1 & -1 & -1 & 4 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} -18 \\ -90 \\ -69 \\ 138 \end{bmatrix}$$

We want to find $\mathbf{x}$ so that $\mathbf{A}\mathbf{x} = \mathbf{b}$. Suppose we have

$$\mathbf{Q} = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}, \qquad \mathbf{R} = \begin{bmatrix} 2 & 4 & 6 & 8 \\ 0 & 6 & 6 & -2 \\ 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

Verify $\mathbf{A} = \mathbf{Q}\mathbf{R}$

What is $\mathbf{Q}^{\mathsf{T}}\mathbf{Q}$? What does this tell you about $\mathbf{Q}^{-1}$?

Use these facts to solve $\mathbf{A}\mathbf{x} = \mathbf{b}$.