Worksheet 2   Numerical Analysis Spring 2023

Name: _____   NetID: ☐☐☐☐☐☐☐☐☐☐☐☐

Name: _____   NetID: ☐☐☐☐☐☐☐☐☐☐☐☐

Name: _____   NetID: ☐☐☐☐☐☐☐☐☐☐☐☐

Name: _____   NetID: ☐☐☐☐☐☐☐☐☐☐☐☐

Work in groups of at least 2 and at most 4.

Problem 1. Define $N = 2^{32}$ equally spaced numbers between a and b by

$$z_i = a + (b-a)\frac{i}{N-1}, \qquad i = 0, 1, \ldots, N-1.$$

- Suppose we want to represent numbers as large as $\pm 1,000,000,000$. What is the smallest the spacing between consecutive z_i can be?

- Suppose we want to be able to represent numbers to 8 decimal points; i.e. to be able to differentiate between 3.14159265 and 3.14159264. What is the largest $b - a$ can be?

Problem 2.

- Convert $(1.110101)_2$ to decimal.

- What is $(1.110101)_2 \times 2^2$ in base 2?

Problem 3.

- Fill out the number line with the numbers $(1.000)_2, (1.001)_2, (1.010)_2, ..., (1.111)_2$.

- Add $(1.b_0b_1b_2)_2 \times 2$ and $(1.b_0b_1b_2)_2 \times \frac{1}{2}$ for all $b_0, b_1, b_2 \in \{0, 1\}$.
- How might you use this to derive a number system which can simultaneously represent numbers as big as $1,000,000,000$ and numbers like π to 8 decimal points?