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Quiz 4   Numerical Analysis Spring 2023

Name: _____   NetID: □□□□□□□□□□□□

Problem 1. Define

$$\mathbf{U} = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{3} \\ 1/\sqrt{3} & 0 \\ 1/\sqrt{3} & -1/\sqrt{3} \\ 0 & -1/\sqrt{3} \end{bmatrix}, \qquad \mathbf{a} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

1. Verify that the columns of $\mathbf{U}$ are orthonormal.   (4pts)

2. Compute $\text{proj}_{U^\perp}(\mathbf{a})$, the projection of $\mathbf{a}$ onto the orthogonal compliment of $\mathbf{U}$. You can use any algorithm/method you like, just show your work. (11pts)

For reference:

$$\mathbf{U} = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{3} \\ 1/\sqrt{3} & 0 \\ 1/\sqrt{3} & -1/\sqrt{3} \\ 0 & -1/\sqrt{3} \end{bmatrix}, \qquad \mathbf{a} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$