



## Quiz 2

## Numerical Analysis Spring 2023

Name: \_\_\_\_\_

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**Problem 1.** Let  $\mathbb{F}$  denote some discrete set of numbers, and suppose that for some  $\epsilon > 0$  the function  $\text{rd} : \mathbb{R} \rightarrow \mathbb{F}$  satisfies

$$|x - \text{rd}(x)| < \epsilon|x|, \quad \forall x \in \mathbb{R}.$$

Find the largest value of  $\epsilon$  for which we can guarantee  $\text{rd}(10^5 + 1) \neq 10^5$ . (4pts)

For this value of  $\epsilon$ , can we guarantee  $\text{rd}(10^{-6} - 10^{-10}) \neq 10^{-6}$ ? Answer yes or no, no justification needed. (1pts)

**Problem 2.** Consider the following problem/task: You are given a differentiable function  $h : [-1, 1] \rightarrow \mathbb{R}$  and must return  $h'(0)$ .

Example inputs/outputs:

input	solution
$h(s) = 1$	0
$h(s) = s^2 + 2s$	2
$h(s) = \sin(s)$	1

Define two inputs  $h$  and  $\tilde{h}$  as near if  $d(h, \tilde{h}) := \max_{s \in [-1, 1]} |h(s) - \tilde{h}(s)|$  is small.

Give a reasonable mathematical definition for the condition number of this problem at an input  $h$ . (3pts)

Decide whether this problem is well-conditioned or not. If it is, explain why. If it is not, provide an example showing that it is not (with justification). (7pts)