Homework 7 — Numerical Analysis Spring 2023

Instructions:

- Due 05/04 at 11:59pm on Gradescope.
- Write the names of anyone you work with on the top of your assignment. If you worked alone, write that you worked alone.
- Show your work.
- Include all code you use as copyable `monospaced` text in the PDF (i.e. not as a screenshot).
- Do not put the solutions to multiple problems on the same page.
- Tag your responses on gradescope. Each page should have a *single* problem tag. Improperly tagged responses will not receive credit.

Problem 1. Let $f(x) = |x|$ and define $p_k(x)$ as the degree k polynomial interpolate to $f(x)$ at the $k+1$ Chebyshev nodes.

(a) For $k = 10$ make one plot with x vs $f(x)$ and x vs $p_k(x)$ and one plot over x vs $|f(x) - p_k(x)|$ with the vertical axis on a log-scale.

(b) Repeat this for $k = 40$

Problem 2. (a) Prove that on a log-log plot, x vs x^k is a line. What is the slope?

(b) What will the plot of x vs $5x^2+3x+1$ look like on a log-log plot when x is large?

(c) Prove that on a log-y plot, x vs ρ^x is a line. What is the slope?

Problem 3. (a) Let $f(x) = \exp(-x)$ and define $p_k(x)$ as the degree k polynomial interpolate to $f(x)$ at the $k+1$ Chebyshev nodes.

Make a plot of k vs

$$\max_{x\in[-1,1]} |f(x) - p_k(x)|$$

for $k = 0, 1, 2, \ldots 20$. Put the y-axis on a log scale and label the axes/plot/etc.

To approximate

$$\max_{x\in[-1,1]} |f(x) - p_k(x)|$$

you can instead take the maximum over 1000 equally spaced points in $[-1, 1]$ and use this instead.

(b) Repeat this for $f(x) = 1/(1 + 16x^2)$ and $k = 0, 1, \ldots, 100$.

(c) Repeat this for $f(x) = |\sin(5x)|^3 = (\sin(5x)^2)^{3/2}$ and $k = 0, 1, \ldots, 100$, but put both axes on log-scales.

Add a line k vs k^{-v}, where v is the largest value so that the $(v-1)$-st derivative of $f(x)$ is continuous.

Problem 4. Let $f(x) = 1/(1+16x^2)$.

For any non-negative integer k, set $n = k^2 + 1$ and let $x_1, \dots, x_n$ be n equally spaced points from -1 to 1 and let $q_k(x)$ be the degree k polynomial minimizing

$$\min_{\deg(q)=k} \sum_{i=1}^{n} (f(x_i) - q(x_i))^2.$$

On a log-y plot, plot the error

$$\max_{x\in[-1,1]} |f(x) - q_k(x)|$$

for $k = 0, 1, \dots, 100$. Add to this plot the error of the Chebyshev interpolant that you computed in 3(b).