# Homework 1 — Numerical Analysis Spring 2023

**Instructions:**

- Due 02/02 at 11:59pm on Gradescope.
- Write the names of anyone you work with on the top of your assignment. If you worked alone, write that you worked alone.
- Show your work.
- Include all code you use as copyable `monospaced` text in the PDF (i.e. not as a screenshot).
- Do not put the solutions to multiple problems on the same page.
- Tag your responses on gradescope. Each page should have a *single* problem tag. Improperly tagged responses will not receive credit.

**Problem 1.**

(a) The Summit supercomputer can do $10^{18}$ floating point operations per second. The supercomputer is physically large. Suppose that the furthest nodes are separated by 100 meters. Roughly how many floating point operations could be done in the time it takes to send a signal from one of these node to the other? (You can assume the signal travels at the speed of light)

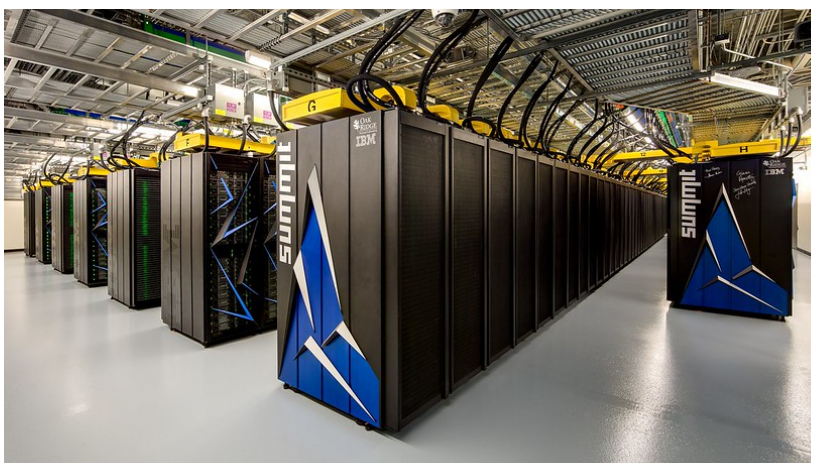


Image of Summit from ORNL.

(b) Suppose $\mathbf{A}$ is both an upper triangular matrix and a lower triangular matrix. Show that $\mathbf{A}$ is diagonal.

(c) Suppose $\mathbf{A}$ is a matrix. Show that $\mathbf{A}^\mathsf{T}\mathbf{A}$ is symmetric.

(d) Suppose $\mathbf{A}$ is a diagonal matrix and $\mathbf{B}$ is upper triangular. Show that $\mathbf{AB}$ is upper triangular.

**Problem 2.** Consider linear equations $f(x) = ax + b$ and $\tilde{f}(x) = ax + (b + \epsilon)$, where $a$ and $b$ are constants and $\epsilon = 10^{-5}$.

(a) Find the solutions to $f(x) = 0$ and $\tilde{f}(x) = 0$ in terms of $a$ and $b$.

(b) Find values of $a$ and $b$ so that the solutions to these two equations are very different. Here "very different" means the solutions differ by much more than $\epsilon$.

(c) Using matplotlib, plot the functions $f(x)$ and $\tilde{f}(x)$ corresponding to your values of $a$ and $b$. Make sure the plots are labeled, and that the axes are chosen such that the two lines can be clearly differentiated from one another and the zeros are clearly visible.

**Problem 3.** Define

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 & -1 & 4 \\ -7 & 0 & 1 \\ 3 & 4 & -3 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 3 \\ 2 \\ -2 \end{bmatrix}.$$

(a) Compute $\mathbf{AB}$.

(b) Compute $\mathbf{Bx}$.

(c) Compute $\mathbf{A}(\mathbf{Bx})$.

(d) Compute $(\mathbf{AB})\mathbf{x}$.

Show your work at each step.

**Problem 4.**

(a) Make the matrices $\mathbf{A}$, $\mathbf{B}$, and vector $\mathbf{x}$ in numpy.

(b) Verify each of the answers you generated in Problem 3 using numpy.