Quiz 5a

Numerical Analysis Fall 2024

Name: ____________________ NetID: | | | | | | | | |

Do not begin until instructed.

Problem 1 (5pts). For a unit vector $\mathbf{q}$, recall $\text{proj}_{\mathbf{q}}(\mathbf{z}) = \mathbf{q}\mathbf{q}^\top\mathbf{z}$. Define

$$\mathbf{q}_1 = \frac{1}{\sqrt{3}}\begin{bmatrix}1\\1\\0\\0\\1\end{bmatrix}, \quad \mathbf{q}_2 = \frac{1}{\sqrt{3}}\begin{bmatrix}0\\1\\1\\0\\-1\end{bmatrix}, \qquad \mathbf{x} = \begin{bmatrix}3\\3\\3\\3\\3\end{bmatrix}.$$

Note that $\mathbf{q}_1$ and $\mathbf{q}_2$ are orthonormal (you do not need to verify this).

Compute the value of $\text{proj}_{\mathbf{q}_2}\left(\mathbf{x} - (\mathbf{q}_1^\top\mathbf{x})\mathbf{q}_1\right)$. *Hint:* What is $\text{proj}_{\mathbf{q}_2}\left(\mathbf{q}_1\right)$?

$$\text{proj}_{\mathbf{q}_2}\left(\mathbf{x} - (\mathbf{q}_1^\top\mathbf{x})\mathbf{q}_1\right) = \begin{bmatrix}\_\_\_\_\\\_\_\_\_\\\_\_\_\_\\\_\_\_\_\\\_\_\_\_\end{bmatrix}$$

Problem 2 (5pts). Consider a QR factorization

$$\begin{bmatrix} | & | & | & | \\ \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 & \mathbf{a}_4 \\ | & | & | & | \end{bmatrix} = \begin{bmatrix} | & | & | & | \\ \mathbf{q}_1 & \mathbf{q}_2 & \mathbf{q}_3 & \mathbf{q}_4 \\ | & | & | & | \end{bmatrix} \begin{bmatrix} 3 & 2 & -1 & 4 \\ 0 & 4 & 2 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

where $\mathbf{q}_1, \dots, \mathbf{q}_4$ are orthonormal.

Write $\mathbf{a}_3$ as a linear combination of $\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3, \mathbf{q}_4$

$\mathbf{a}_3 = \boxed{}\,\mathbf{q}_1 + \boxed{}\,\mathbf{q}_2 + \boxed{}\,\mathbf{q}_3 + \boxed{}\,\mathbf{q}_4$

Problem 3 (5pts). Suppose $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$. In big-$O$, how many operations does it take to compute $\mathbf{u}\mathbf{u}^\mathsf{T}\mathbf{v}$ when the order of operations are done in the following ways:

$(\mathbf{u}\mathbf{u}^\mathsf{T})\mathbf{v}$: $O\left(\boxed{}\right)$ flops

$\mathbf{u}(\mathbf{u}^\mathsf{T}\mathbf{v})$: $O\left(\boxed{}\right)$ flops