Quiz 2a

Numerical Analysis Fall 2024

Name: ____________________

NetID: | | | | | | | | | |

Do not begin until instructed.

Problem 1 (5pts). Define

$$\mathbf{M} = \begin{bmatrix} 1 & 3 & 1 & 2 \\ 0 & 3 & -1 & 4 \\ 2 & 5 & -2 & 4 \\ -2 & 1 & 3 & 1 \end{bmatrix}, \quad \mathbf{m}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \\ -2 \end{bmatrix}, \quad \mathbf{m}_2 = \begin{bmatrix} 3 \\ 3 \\ 5 \\ 1 \end{bmatrix}, \quad \mathbf{m}_3 = \begin{bmatrix} 1 \\ -1 \\ -2 \\ 3 \end{bmatrix}, \quad \mathbf{m}_4 = \begin{bmatrix} 2 \\ 4 \\ 4 \\ 1 \end{bmatrix}$$

Note that

$$\mathbf{M}^\mathsf{T}\mathbf{M} = \begin{bmatrix} 9 & 11 & -9 & 8 \\ 11 & 44 & -7 & 39 \\ -9 & -7 & 15 & -7 \\ 8 & 39 & -7 & 37 \end{bmatrix}.$$

Evaluate the following:

$\mathbf{m}_1^\mathsf{T}\mathbf{m}_2 =$ ☐ $\quad$ $\mathbf{m}_1^\mathsf{T}\mathbf{m}_4 =$ ☐ $\quad$ $\mathbf{m}_2^\mathsf{T}\mathbf{m}_3 =$ ☐

$\mathbf{m}_2^\mathsf{T}\mathbf{m}_1 =$ ☐ $\quad$ $\mathbf{m}_1^\mathsf{T}\mathbf{m}_1 =$ ☐ $\quad$ $\|\mathbf{m}_1\| =$ ☐

Problem 2 (10pts). Suppose that $\mathbf{A}$ is such that

$$\mathbf{A}^\mathsf{T}\mathbf{A} = \begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} x \\ y \end{bmatrix}.$$

a) Write $\|\mathbf{v}\|$ in terms of x and y.

b) Write $\|\mathbf{Av}\|$ in terms of x and y. *Hint*: How do $\|\mathbf{u}\|$ and $\mathbf{u}^\top\mathbf{u}$ relate?

c) Find a choice of x and y so that $\|\mathbf{Av}\| = 2\|\mathbf{v}\|$.

d) Prove that no matter how we pick x and y that $\|\mathbf{Av}\| \leq 2\|\mathbf{v}\|$.