Quiz 1a

Numerical Analysis Fall 2024

Name: ____________________ NetID:

Do not begin until instructed.

Problem 1 (9pts). Let

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & -4 \\ 1 & -2 & 3 \\ -3 & 1 & 1 \\ 2 & -4 & 6 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{Z} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 1 & 0 \end{bmatrix}.$$

Compute the following:

$$\mathbf{Ax} = \begin{bmatrix} \; \\ \; \\ \; \\ \; \end{bmatrix}, \quad \mathbf{Ay} = \begin{bmatrix} \; \\ \; \\ \; \\ \; \end{bmatrix}, \quad \mathbf{AZ} = \begin{bmatrix} \; & \; \\ \; & \; \\ \; & \; \\ \; & \; \end{bmatrix}$$

Problem 2 (6pts). Recall the definition of linear dependence:

> A collection $\mathbf{v}_1, \dots, \mathbf{v}_n$ of vectors is *linearly dependent* if there exist scalars $c_1, \dots, c_n$, not all zero, such that $c_1\mathbf{v}_1 + \cdots + c_n\mathbf{v}_n = \mathbf{0}$.

Define

$$\mathbf{a}_1 = \begin{bmatrix} 2 \\ 1 \\ -3 \\ 2 \end{bmatrix}, \quad \mathbf{a}_2 = \begin{bmatrix} 1 \\ -2 \\ 1 \\ -4 \end{bmatrix}, \quad \mathbf{a}_3 = \begin{bmatrix} -4 \\ 3 \\ 1 \\ 6 \end{bmatrix}.$$

Prove that $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$ are linearly dependent.

Hint: how do $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$ relate to the matrix $\mathbf{A}$ from the previous problem?