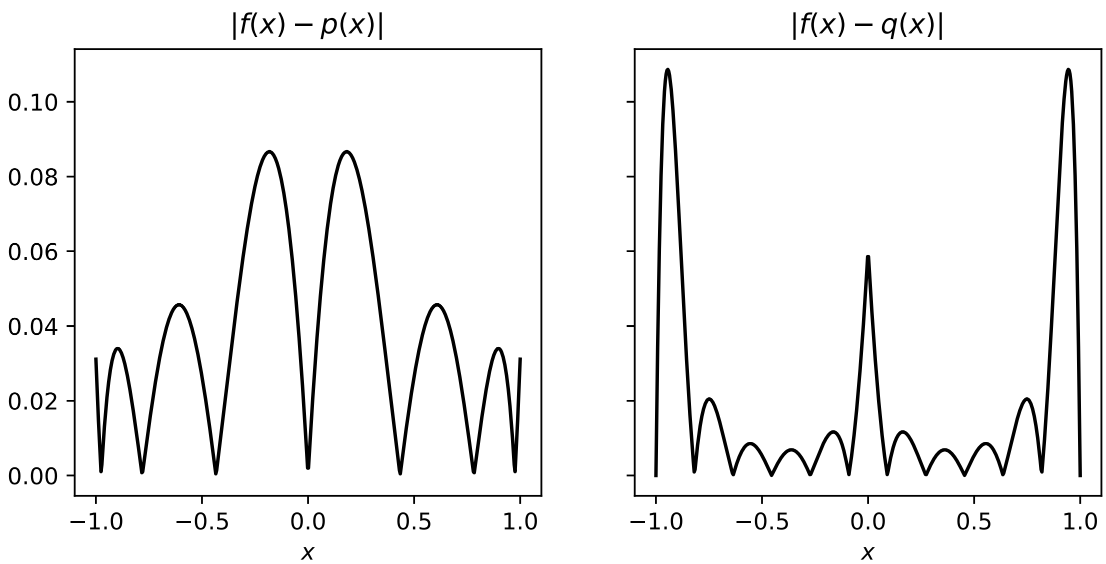
Quiz 7 — Numerical Analysis Fall 2023

Name: ______________________ NetID: [| | | | | | | | | |]

Do not begin until instructed. Clearly justify each step. Indicate your answer by filling in the blanks in the matrices below (you can leave zeros blank or write them in).

Problem 1. Here $f(x) = |x|$ and $p(x)$ and $q(x)$ are polynomial interpolants to $f(x)$.

The two interpolants are of different degrees, and one uses uniformly spaced interpolation nodes while the other uses Chebyshev nodes.



(a) Which of $p(x)$ or $q(x)$ is using Chebyshev nodes? [ ]

(b) What is the degree of $p(x)$? [ ]

(c) What is the degree of $q(x)$? [ ]

Problem 2. Suppose **A** has and SVD $\mathbf{U}\mathbf{\Sigma}\mathbf{V}^\top$ where,

$$\mathbf{A} = \begin{bmatrix} | & | & & | \\ \mathbf{u}_1 & \mathbf{u}_2 & \cdots & \mathbf{u}_n \\ | & | & & | \end{bmatrix} \begin{bmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \ddots & \\ & & & \sigma_n \end{bmatrix} \begin{bmatrix} | & | & & | \\ \mathbf{v}_1 & \mathbf{v}_2 & \cdots & \mathbf{v}_n \\ | & | & & | \end{bmatrix}^\top .$$

Define the block matrix:

$$\mathbf{B} = \begin{bmatrix} \mathbf{0} & \mathbf{A} \\ \mathbf{A}^\top & \mathbf{0} \end{bmatrix}.$$

Show that

$$\mathbf{x} = \begin{bmatrix} \mathbf{u}_i \\ -\mathbf{v}_i \end{bmatrix}$$

is an eigenvector of **B**. What is the corresponding eigenvalue?