Quiz 6

Numerical Analysis Fall 2023

Name: ____________________ NetID: | | | | | | | | | | | |

Do not begin until instructed. Clearly justify each step. Indicate your answer by filling in the blanks in the matrices below (you can leave zeros blank or write them in).

Problem 1. Suppose $\mathbf{A}$ has and SVD $\mathbf{U}\mathbf{\Sigma}\mathbf{V}^{\mathsf{T}}$ where,

$$\mathbf{A} = \begin{bmatrix} | & | & & | \\ \mathbf{u}_1 & \mathbf{u}_2 & \cdots & \mathbf{u}_n \\ | & | & & | \end{bmatrix} \begin{bmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \ddots & \\ & & & \sigma_n \end{bmatrix} \begin{bmatrix} | & | & & | \\ \mathbf{v}_1 & \mathbf{v}_2 & \cdots & \mathbf{v}_n \\ | & | & & | \end{bmatrix}^{\mathsf{T}}.$$

Show that $\mathbf{v}_2$ is an eigenvectors of $\mathbf{A}^{\mathsf{T}}\mathbf{A}$. What is the corresponding eigenvalue?

Problem 2. Suppose $\mathbf{A}$ has eigenvalue decomposition $\mathbf{V}\mathbf{\Lambda}\mathbf{V}^\mathsf{T}$ where

$$\mathbf{\Lambda} = \begin{bmatrix} 6 & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & 5 & \cdot \\ \cdot & \cdot & \cdot & 3 \end{bmatrix} \quad \text{and} \quad \mathbf{V} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 & \cdot & \cdot \\ 1 & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & 1 \\ \cdot & \cdot & -1 & 1 \end{bmatrix}, \quad \text{and let} \quad \mathbf{x} = \mathbf{V} \begin{bmatrix} -4 \\ 3 \\ -2 \\ 1 \end{bmatrix}.$$

1. Write an eigendecomposition $\mathbf{U}\widetilde{\mathbf{\Lambda}}\mathbf{U}^\mathsf{T}$ for $(2\mathbf{A} - 5\mathbf{I})^{-1}$:

$$\mathbf{U} = \begin{bmatrix} \; & \; & \; & \; \\ \; & \; & \; & \; \\ \; & \; & \; & \; \\ \; & \; & \; & \; \end{bmatrix}, \qquad \widetilde{\mathbf{\Lambda}} = \begin{bmatrix} \; & \; & \; & \; \\ \; & \; & \; & \; \\ \; & \; & \; & \; \\ \; & \; & \; & \; \end{bmatrix}$$

2. Let $\mathbf{z}_k = \left((2\mathbf{A} - 5\mathbf{I})^{-1}\right)^k \mathbf{x}$. What is $\lim_{k\to\infty} \frac{\mathbf{z}_k}{\|\mathbf{z}_k\|}$?

$$\lim_{k\to\infty} \frac{\mathbf{z}_k}{\|\mathbf{z}_k\|} = \begin{bmatrix} \; \\ \; \\ \; \\ \; \end{bmatrix}$$