



## Quiz 5

## Numerical Analysis Fall 2023

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Name: \_\_\_\_\_

NetID: 

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You can work with others on the quiz, but you must indicate anyone who you work with. Each person should turn in their own quiz, and you should write inside the boxes (either print and scan or use a tablet).

Clearly justify each step. Because this has a longer time limit, the grading standards will be higher than other quizzes.

**Problem 1.** Suppose  $\mathbf{W}$  is an orthogonal matrix ( $\mathbf{W}^T \mathbf{W} = \mathbf{I}$ ). Let  $\mathbf{z}$  be a vector in the span of the columns of  $\mathbf{W}$ . Prove  $\text{proj}_{\mathbf{W}}(\mathbf{z}) = \mathbf{z}$  and  $\text{proj}_{\mathbf{W}^\perp}(\mathbf{z}) = \mathbf{0}$ .

**Problem 2.** Define

$$\mathbf{x} = \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}, \quad \mathbf{W} = \frac{1}{\sqrt{3}} \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \text{ and note that: } \text{proj}_{\mathbf{W}}(\mathbf{x}) = \begin{bmatrix} 3 \\ 4 \\ 1 \\ 1 \end{bmatrix}$$

Now, define

$$\mathbf{y} = \mathbf{x} + 12 \begin{bmatrix} -1 \\ -1 \\ 0 \\ 1 \end{bmatrix} - 2 \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}.$$

Use Problem 1 to compute  $\text{proj}_{\mathbf{W}^\perp}(\mathbf{y})$ , the projection of  $\mathbf{y}$  onto the orthogonal complement of  $\mathbf{W}$ .

$$\text{proj}_{\mathbf{W}^\perp}(\mathbf{y}) = \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \end{bmatrix}$$