Quiz 2 — Numerical Analysis Fall 2023

Name: _____ NetID: | | | | | | | | | | | |

Do not begin until instructed. Clearly justify each step. Circle your final answer.

Problem 1 (7pts)**.** Define

$$\mathbf{M} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \qquad \mathbf{X} = \begin{bmatrix} 1 & 1 & 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 & 1 & 1 \end{bmatrix}$$

a) What is $\mathbf{M}^\mathsf{T}\mathbf{M}$?

b) What is $\|\mathbf{MX}\|_\mathsf{F}$? *Hint*: Use part a) and properties of the Frobenius norm.

Problem 2 (8pts). Suppose that $\mathbf{A}$ has an SVD $\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^\mathsf{T}$ given by

$$\mathbf{U} = \begin{bmatrix} -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{bmatrix}, \quad \mathbf{\Sigma} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \quad \mathbf{V}^\mathsf{T} = \begin{bmatrix} \frac{1}{\sqrt{6}} & \sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{7\sqrt{3}} & \frac{5}{7\sqrt{3}} & \frac{11}{7\sqrt{3}} \\ \frac{9}{7\sqrt{2}} & -\frac{2\sqrt{2}}{7} & \frac{1}{7\sqrt{2}} \end{bmatrix}.$$

Define

$$\mathbf{x} = \begin{bmatrix} \frac{9}{7\sqrt{2}} \\ -\frac{2\sqrt{2}}{7} \\ \frac{1}{7\sqrt{2}} \end{bmatrix}, \qquad \mathbf{y} = \begin{bmatrix} \frac{1}{\sqrt{6}} \\ \sqrt{\frac{2}{3}} \\ -\frac{1}{\sqrt{6}} \end{bmatrix}.$$

a) What is $\mathbf{A}\mathbf{x}$? *Hint*: Note that $\mathbf{x}$ is the last column of $\mathbf{V}$ ($\mathbf{x}^\mathsf{T}$ is the bottom row of $\mathbf{V}^\mathsf{T}$).

b) What is $\|\mathbf{A}\mathbf{y}\|_2$? *Hint*: Note that $\mathbf{y}$ is the first column of $\mathbf{V}$ ($\mathbf{y}^\mathsf{T}$ is the top row of $\mathbf{V}^\mathsf{T}$).