Quiz 1 (bonus)

Numerical Analysis Fall 2023

Name: ______________________

NetID:

You can earn back up to half the lost points from quiz 1.

Problem 1. A matrix $\mathbf{T}$ is said to be *tridiagonal* if $[\mathbf{T}]_{i,j} = 0$ for all i, j where $|i - j| > 1$.

Let $\mathbf{A}$ be a $n \times n$ tridiagonal matrix and $\mathbf{D}$ a $n \times n$ diagonal matrix. Prove that $\mathbf{AD}$ is tridiagonal.

Hint: for $n \times n$ matrices $\mathbf{X}, \mathbf{Y}$, $[\mathbf{XY}]_{i,j} = \sum_{k=1}^{n} [\mathbf{X}]_{i,k} [\mathbf{Y}]_{k,j}$.

Problem 2. Suppose that

$$\begin{bmatrix} | & | \\ \mathbf{x}_1 & \mathbf{x}_2 \\ | & | \end{bmatrix} \begin{bmatrix} \pi & e \\ \pi^2 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, \qquad \begin{bmatrix} | & | & | \\ \mathbf{y}_1 & \mathbf{y}_2 & \mathbf{y}_3 \\ | & | & | \end{bmatrix} \begin{bmatrix} 8 & 2 \\ 2 & 1 \\ -3 & 3 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 1 \\ 1 & -1 \\ 0 & 1 \end{bmatrix}$$

Compute the following:

$$\text{a)} \quad \begin{bmatrix} | & | \\ \mathbf{x}_1 & \mathbf{x}_2 \\ | & | \end{bmatrix} \begin{bmatrix} \pi - e \\ \pi^2 + 3 \end{bmatrix}, \qquad \text{b)} \quad \begin{bmatrix} | & | & | & | & | \\ \mathbf{x}_1 & \mathbf{y}_1 & \mathbf{y}_2 & \mathbf{y}_3 & \mathbf{x}_2 \\ | & | & | & | & | \end{bmatrix} \begin{bmatrix} -\pi & 2e \\ 8 & 12 \\ 2 & 4 \\ -3 & 3 \\ -\pi^2 & -6 \end{bmatrix}$$

a)

b)