



# Quiz 1

# Numerical Analysis Fall 2023

Name: \_\_\_\_\_

NetID: 

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Do not begin until instructed. Circle your final answer.

**Problem 1** (5pts). A matrix  $\mathbf{T}$  is said to be *tridiagonal* if  $[\mathbf{T}]_{i,j} = 0$  for all  $i, j$  where  $|i - j| > 1$ .

Let  $\mathbf{A}$  be a  $n \times n$  tridiagonal matrix and  $\mathbf{D}$  a  $n \times n$  diagonal matrix. Prove that  $\mathbf{AD}$  is tridiagonal.

*Hint:* for  $n \times n$  matrices  $\mathbf{X}, \mathbf{Y}$ ,  $[\mathbf{XY}]_{i,j} = \sum_{k=1}^n [\mathbf{X}]_{i,k} [\mathbf{Y}]_{k,j}$ .

**Problem 2** (10pts). Note that

$$\begin{bmatrix} 795/68 & 154/17 \\ 120/17 & 84/17 \\ 120/17 & 84/17 \\ 315/68 & 70/17 \end{bmatrix} \begin{bmatrix} -4/5 & 2/3 \\ 8/7 & -3/4 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \begin{bmatrix} 13/4 & -2 & 479/6 \\ 3/4 & 0 & 25/2 \\ -14/3 & -2/4 & -1016/9 \\ 2/3 & -1 & 158/9 \end{bmatrix} \begin{bmatrix} 14/3 \\ -2/5 \\ -1/5 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

Compute the following:

$$\text{a) } \begin{bmatrix} 795/68 & 154/17 \\ 120/17 & 84/17 \\ 120/17 & 84/17 \\ 315/68 & 70/17 \end{bmatrix} \begin{bmatrix} (-4/5 + 2/3) \\ (8/7 - 3/4) \end{bmatrix}, \quad \text{b) } \begin{bmatrix} 795/68 & 13/4 & -2 & 479/6 & 154/17 \\ 120/17 & 3/4 & 0 & 25/2 & 84/17 \\ 120/17 & -14/3 & -2/4 & -1016/9 & 84/17 \\ 315/68 & 2/3 & -1 & 158/9 & 70/17 \end{bmatrix} \begin{bmatrix} 4/5 & 8/3 \\ 14/3 & 0 \\ -2/5 & 0 \\ -1/5 & 0 \\ -8/7 & -3 \end{bmatrix}$$

$\begin{matrix} a_1 & a_2 & b_1 & b_2 & b_3 & b_4 & b_5 \end{matrix}$

a)

b)