Homework 1 — Numerical Analysis Fall 2023

Instructions:

- Due 09/15 at 5:00pm on Gradescope.
- You must follow the submission policy in the syllabus

Problem 1.

(a) The Summit supercomputer can do 10^{18} floating point operations per second. The supercomputer is physically large, consisting of many different nodes (individual computers) connected together. Suppose that the furthest nodes are separated by 100 meters. Roughly how many floating point operations could be done in the time it takes to send a signal from one of these node to the other? (You can assume the signal travels at the speed of light)

(b) What consequences does your answer in (a) have for supercomputing?

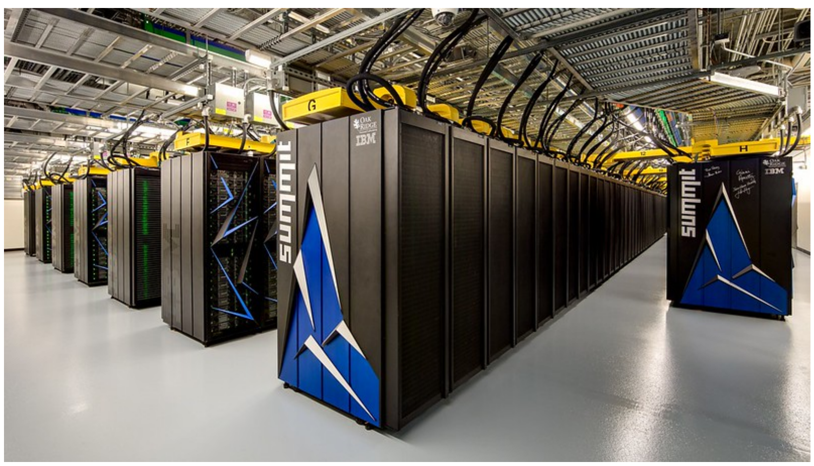

Image of Summit from ORNL.

Problem 2.

(a) Suppose $\mathbf{A}$ is both an upper triangular matrix and a lower triangular matrix. Show that $\mathbf{A}$ is diagonal.

(b) Suppose $\mathbf{A}$ is a matrix. Show that $\mathbf{A}^\mathsf{T}\mathbf{A}$ is symmetric.

(c) Suppose $\mathbf{A}$ is a diagonal matrix and $\mathbf{B}$ is upper triangular. Show that $\mathbf{AB}$ is upper triangular.

(d) Find an example of an orthogonal matrix $\mathbf{A}$ (i.e. $\mathbf{A}^\mathsf{T}\mathbf{A} = \mathbf{I}$) for which $\mathbf{A}\mathbf{A}^\mathsf{T} \neq \mathbf{I}$.

Problem 3. Define

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 & -1 & 4 \\ -7 & 0 & 1 \\ 3 & 4 & -3 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 3 \\ 2 \\ -2 \end{bmatrix}.$$

(a) Compute $\mathbf{AB}$.
(b) Compute $\mathbf{Bx}$.
(c) Compute $\mathbf{A}(\mathbf{Bx})$.
(d) Compute $(\mathbf{AB})\mathbf{x}$.

Show your work at each step.

Problem 4.

(a) Make the matrices $\mathbf{A}$, $\mathbf{B}$, and vector $\mathbf{x}$ in numpy.
(b) Verify each of the answers you generated in Problem 3 using numpy.

Problem 5. Consider linear equations $f(x) = ax + b$ and $\tilde{f}(x) = ax + (b + \epsilon)$, where a and b are constants and $\epsilon = 10^{-5}$.

(a) Find the solutions to $f(x) = 0$ and $\tilde{f}(x) = 0$ in terms of a and b.
(b) Find values of a and b so that the solutions to these two equations are very different. Here "very different" means the solutions differ by much more than ϵ.
(c) Using matplotlib, plot the functions $f(x)$ and $\tilde{f}(x)$ corresponding to your values of a and b. Make sure the plots are labeled, and that the axes bounds are chosen such that the two lines can be clearly differentiated from one another and the zeros are clearly visible.