

Homework 5: Mathematical Statistics (MATH-UA 234)

Due 11/03 at the beginning of class on Gradescope

Problem 1. Suppose X_1, \dots, X_n are real numbers. Then the likelihood function is

$$L_n(\theta) = \prod_{i=1}^n f_\theta(X_i).$$

In your own words, describe the meaning of $L_n(\theta)$ for a specific value of θ when

- (a) f_θ is a probability mass function
- (b) f_θ is a probability density function

Problem 2 (Wasserman 9.1). In this problem, you will fill in some pieces we skipped in lecture. If you take the approach of searching for critical points, you should justify why your response is a global maximum.

- (a) Let $X_1, \dots, X_n \sim \text{Ber}(p)$ where p is an unknown parameter. Show $\ell_n(p) = S \log(p) + (n - S) \log(1 - p)$, where $S = X_1 + \dots + X_n$, and then find the maximizer of $\ell_n(p)$.
- (b) Let $X_1, \dots, X_n \sim N(\mu, \sigma^2)$ where μ and σ are unknown parameters. Show $\ell(\mu, \sigma) = c - n \log(\sigma) - nS^2 / (2\sigma^2) - n(\bar{X} - \mu)^2 / (2\sigma^2)$, where $\bar{X} = n^{-1}(X_1 + \dots + X_n)$, $S^2 = n^{-1} \sum_{i=1}^n (X_i - \bar{X})^2$, and c is some constant, and then find the maximizer of $\ell_n(\mu, \sigma)$.

Problem 3 (Wasserman 9.1). Let $X_1, \dots, X_n \sim \text{Gamma}(\alpha, \beta)$ where α and β are unknown parameters. Find the method of moments estimator for α and β .

Problem 4 (Wasserman 9.2). Let $X_1, \dots, X_n \sim \text{Unif}(a, b)$ where a and b are unknown parameters with $a < b$.

- (a) Find the method of moments estimator for a and b .
- (b) Find the maximum likelihood estimators for a and b .

Problem 5 (Wasserman 9.5). Let $X_1, \dots, X_n \sim \text{Poisson}(\lambda)$ where λ is an unknown parameter.

- (a) Find the method of moments estimator for λ .
- (b) Find the maximum likelihood estimators for λ .

Problem 6 (Wasserman 9.6). Let $X_1, \dots, X_n \sim N(\theta, 1)$ where θ is an unknown parameter. Define

$$Y_i = \begin{cases} 1 & X_i > 0 \\ 0 & X_i \leq 0. \end{cases}$$

Let $\psi = \mathbb{P}[Y_1 = 1]$.

Find the maximum likelihood estimator for ψ .

problems with a textbook reference are based on, but not identical to, the given reference