Bayesian Method

- Start w. statistical model $\mathcal{F} = \{F_\theta\}$

- use our knowledge / beliefs to come up w. prior dist $f(\theta)$ for Θ. Here we view Θ as a RV describing our beliefs.

- observe data $X_1, \ldots, X_n$

- update belief about Θ based on data

$$f(\theta \mid X_1, \ldots, X_n) \propto f(X_1, \ldots, X_n \mid \theta) f(\theta)$$

This update is justified if

$X_1, \ldots, X_n \sim F_\Theta$, where Θ is distributed according to our prior

Ex. $X_1, \ldots, X_n \sim \text{Ber}(p)$

$$f(p) = \begin{cases} 1 & 0<p<1 \\ 0 & \text{o.w.} \end{cases} \quad (\text{unif. prior})$$

$$f(x|\theta) = p^x (1-p)^{1-x}$$

$$\mathcal{L}_n(\theta) = \prod_{i=1}^{n} f(X_i|\theta) = \prod_{i=1}^{n} p^{X_i}(1-p)^{1-X_i} = p^S(1-p)^{n-S}$$

$$S = X_1 + \cdots + X_n$$

$$\Rightarrow f(\theta|X_1,\ldots,X_n) \propto f(\theta)\mathcal{L}_n(\theta)$$

$$= 1 \cdot p^S (1-p)^{n-S}$$

Beta(α,β) pdf $\quad x \mapsto \dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$

what are α, β in terms of our problem?

$\alpha = S+1, \quad \beta = n-S+1$