Quiz

$$X_1, \dots, X_n \sim F_\alpha$$

$$F_\alpha(x) = \begin{cases} 1 - \frac{1}{x^\alpha} & x \geq 1 \\ 0 & x < 1 \end{cases}$$

$$\text{pdf} \quad f_\alpha(x) = F_\alpha'(x) = \begin{cases} \alpha x^{-(\alpha+1)} & x \geq 1 \\ 0 & x < 1 \end{cases}$$

$$\mathcal{L}_n(\alpha) = \prod_{i=1}^{n} f_\alpha(X_i) = \prod_{i=1}^{n} \alpha X_i^{-(\alpha+1)} \neq \alpha^n X^{-n(\alpha+1)}$$

$$\ell_n(\alpha) = \log\left(\prod_{i=1}^{n} f_\alpha(X_i)\right) = \sum_{i=1}^{n} \log\left(\alpha X_i^{-(\alpha+1)}\right)$$

$$= \sum_{i=1}^{n} \log(\alpha) + \log\left(X_i^{-(\alpha+1)}\right)$$

$$= \sum_{i=1}^{n} \log(\alpha) - (\alpha+1)\log(X_i)$$

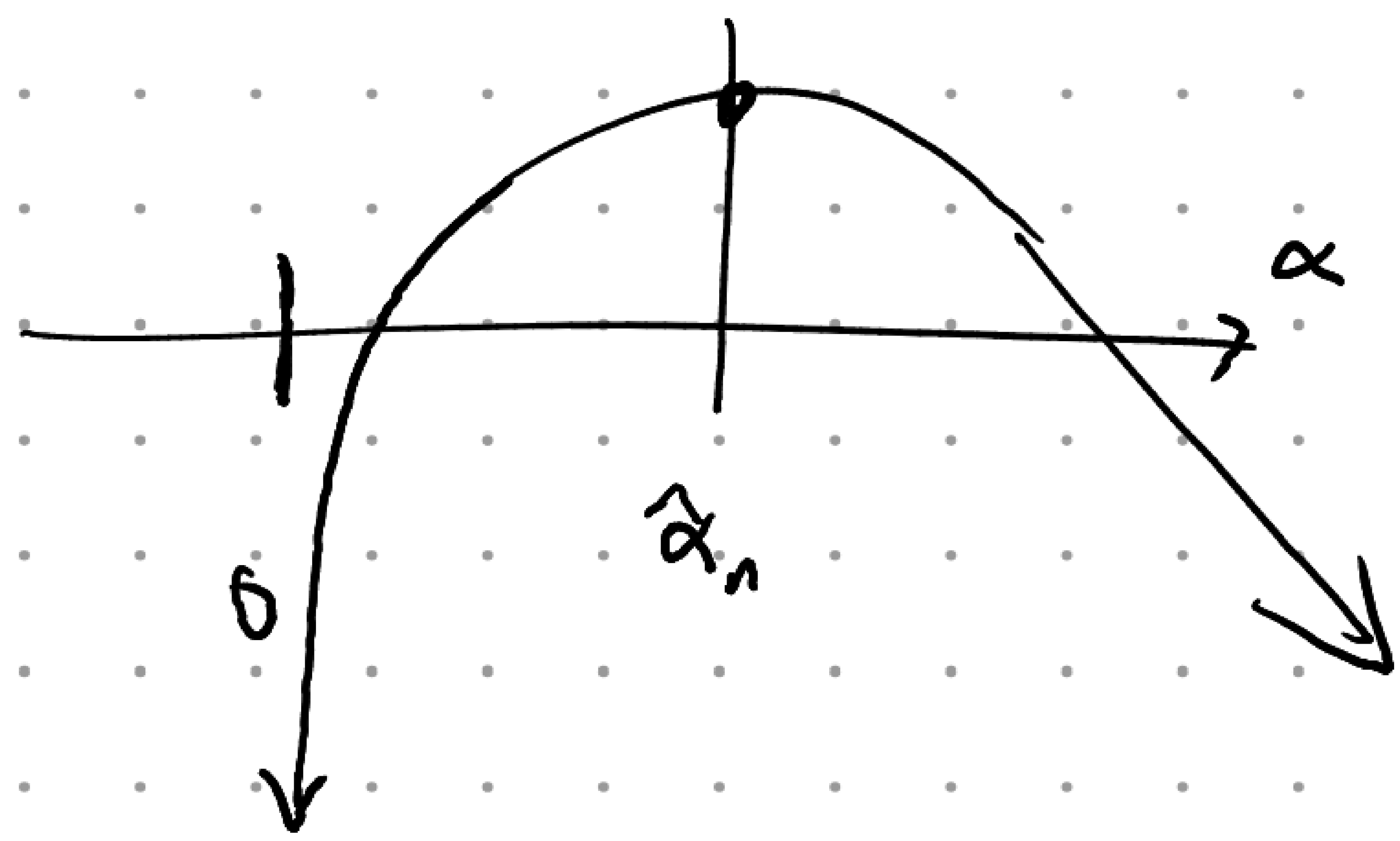
$$= n\log(\alpha) - (\alpha+1)\sum_{i=1}^{n} \log(X_i)$$

$$\ell_n(\alpha) = n\log(\alpha) - (\alpha+1)\sum_{i=1}^{n} \log(X_i)$$

$$\hat{\alpha} = \underset{\alpha > 0}{\operatorname{argmax}}\ \ell_n(\alpha)$$

$\ell_n(0) = -\infty$

$\ell_n(\infty) = -\infty$



$\Rightarrow$ maximizer $\hat{\alpha}$ positive and finite

$$\ell_n'(\alpha) = \frac{n}{\alpha} - \sum_{i=1}^{n} \log(X_i) = 0$$

$$\Rightarrow \alpha = \frac{n}{\sum_{i=1}^{n} \log(X_i)}$$ critical pt.

must be maximum

$$\hat{\alpha} = \frac{n}{\sum_{i=1}^{n} \log(X_i)}$$

Up to now (Frequentist)

- probabilities can be viewed as limiting distribution of repeated experiments

- parameters are fixed, unknown constants

Bayesian

- probability describe degree of belief. (i.e. betting odds) We can now assign probability to events w. no limiting dist.

- we can make probability statements about parameters

- we make inferences by producing a distribution for the parameter.

Allows us to incorporate our knowledge of how the world works

The Bayesian Method

1. Choose prior distribution $f(\theta)$ for Θ

   This reflects our belief about Θ

2. Choose statistical model $f(x|\theta) \to F_\theta$

   This is the same as before, we assume our data comes from this model

3. After we get data $X_1, \ldots, X_n \sim F_\theta$, update our beliefs about the distribution of Θ and compute posterior distribution

   $f(\theta | X_1, \ldots, X_n)$

Bayes Theorem

$$\mathbb{P}[\Theta = \theta \mid X = x] = \frac{\mathbb{P}[X = x, \Theta = \theta]}{\mathbb{P}[X = x]}$$

$$= \frac{\mathbb{P}[X = x \mid \Theta = \theta]\,\mathbb{P}[\Theta = \theta]}{\sum_{\theta'} \mathbb{P}[X = x \mid \Theta = \theta']\,\mathbb{P}[\Theta = \theta']}$$

$$f(\theta \mid x) = \frac{f(x \mid \theta) f(\theta)}{\int f(x \mid \theta') f(\theta') d\theta'}$$

pdf for $F_\theta(X)$ gives prob for sampling X given value of θ

$$\propto f(x \mid \theta) f(\theta)$$

constant not important, just integrable

If we have iid data,

$$f(x_1, \ldots, x_n \mid \theta) = \prod_{i=1}^{n} f(x_i \mid \theta) = \mathcal{L}_n(\theta)$$

$$f(\theta \mid x_1, \ldots, x_n) \propto f(x_1, \ldots, x_n \mid \theta) f(\theta) = \mathcal{L}_n(\theta) f(\theta)$$