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Work in groups of at least 2 and at most 4.

Suppose $X_1, \dots, X_n \sim N(\mu, 1)$ where μ is some unknown fixed parameter. We will assume $\Theta = \mathbb{R}$; i.e. that the mean could be anything.

Our goal is to design and analyze a test for the null hypothesis: *the mean μ is less than or equal to zero.*

Define $T(x_1, \dots, x_n) = n^{-1}(x_1 + \cdots + x_n)$. Our test is the following:

- if $T(X_1, \dots, X_n) > c$: reject the null hypothesis
- if $T(X_1, \dots, X_n) \leq c$: retain the null hypothesis

In other words, the rejection region is $\{(x_1, \dots, x_n) \in \mathbb{R}^n : T(x_1, \dots, x_n) > c\}$.

Last time, we saw the power function is

$$\beta(\mu) = \mathbb{P}[T(X_1, \dots, X_n) > c | X_1, \dots, X_n \sim N(\mu, 1)] = 1 - \Phi(n^{1/2}(c - \mu)),$$

where $\Phi(x) = \mathbb{P}[Z \leq x | Z \sim N(0, 1)]$ is the CDF for a standard normal random variable.

From this, we can conclude that the size of the test is

$$\text{size} = \sup_{\mu \in \Theta_0} \beta(\mu) = \sup_{\mu \leq 0} \beta(\theta) = \beta(0) = 1 - \Phi(n^{1/2}c),$$

and so the test is of level-α if $c \geq n^{-1/2}\Phi^{-1}(1 - \alpha)$.

5. For each α we will pick $c = c_\alpha$ so that our test is a size-α test. Suppose $n = 8$ and we sample data points $\{3, -1, 5, 0, 3, -4, -1, 2\}$. What is the corresponding p-value?

   What if we had the same sample mean, but $n = 100$? What about $n = 1$? Before doing the computation, think about if the p-value should be smaller or larger.

   What if instead we had data $\{0, 2, -1, 1, -2, -3, 4, -1\}$?

   In Wolfram Alpha you can evaluate $\Phi(1.24)$ using the query "probabilities for normal 1.24".

6. Suppose instead, for each α, we pick c twice what we used in the previous part. How do the p-values change?

7. Find a $1-\alpha$ confidence interval (a, b) for μ. That is, find (a, b) such that $\mathbb{P}[\mu \in (a, b)] \geq 1-\alpha$. How do (a, b) relate to c above?