Ex. $X_1, \dots, X_n \sim N(\mu, 1)$

$$\Theta_0 = \{\mu \le 0\}, \quad \Theta_1 = \{\mu > 0\}$$

Test: reject H_0 if $\bar{X}_n > c$

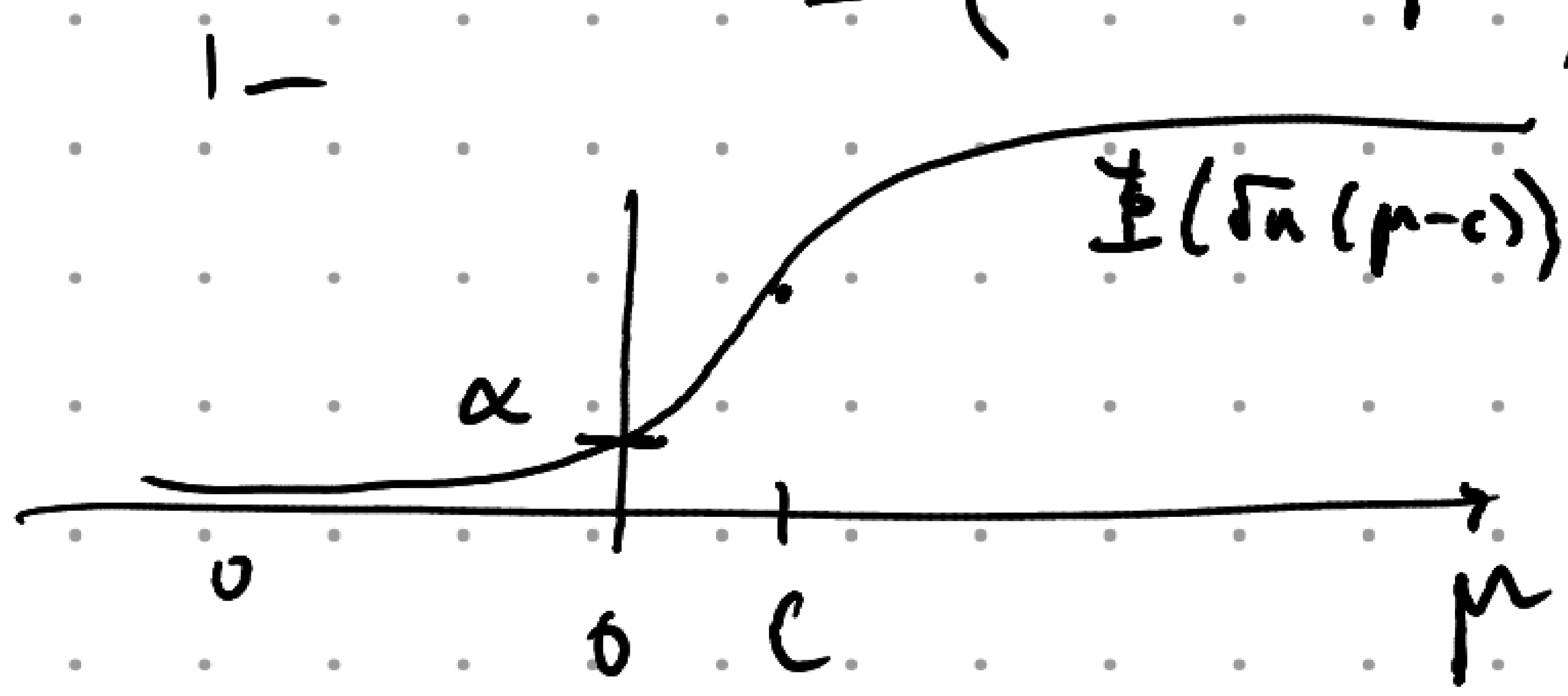
$$R = \left\{ (x_1, \dots, x_n) : \frac{x_1 + \dots + x_n}{n} > c \right\}$$

$$\beta(\mu) = \mathbb{P}\left[\bar{X} > c \mid X_1, \dots, X_n \sim N(\mu, 1)\right]$$

$$= \mathbb{P}\left[\sqrt{n}(\bar{X} - \mu) > \sqrt{n}(c - \mu) \mid X_1, \dots, X_n \sim N(\mu, 1)\right]$$

$$= \mathbb{P}\left[Z \ge \sqrt{n}(c - \mu) \mid Z \sim N(0,1)\right]$$

$$= 1 - \Phi\left(\sqrt{n}(c - \mu)\right) = \Phi\left(\sqrt{n}(\mu - c)\right)$$



$$\text{size} = \sup_{\mu \le 0} \beta(\mu) = \beta(0) = 1 - \Phi(\sqrt{n} \cdot c)$$

β inc. in μ

level α if $c \ge \dfrac{\Phi^{-1}(1-\alpha)}{\sqrt{n}}$

Def. Suppose $\forall \alpha \in (0,1)$ we have a size α test with rejection region R_α

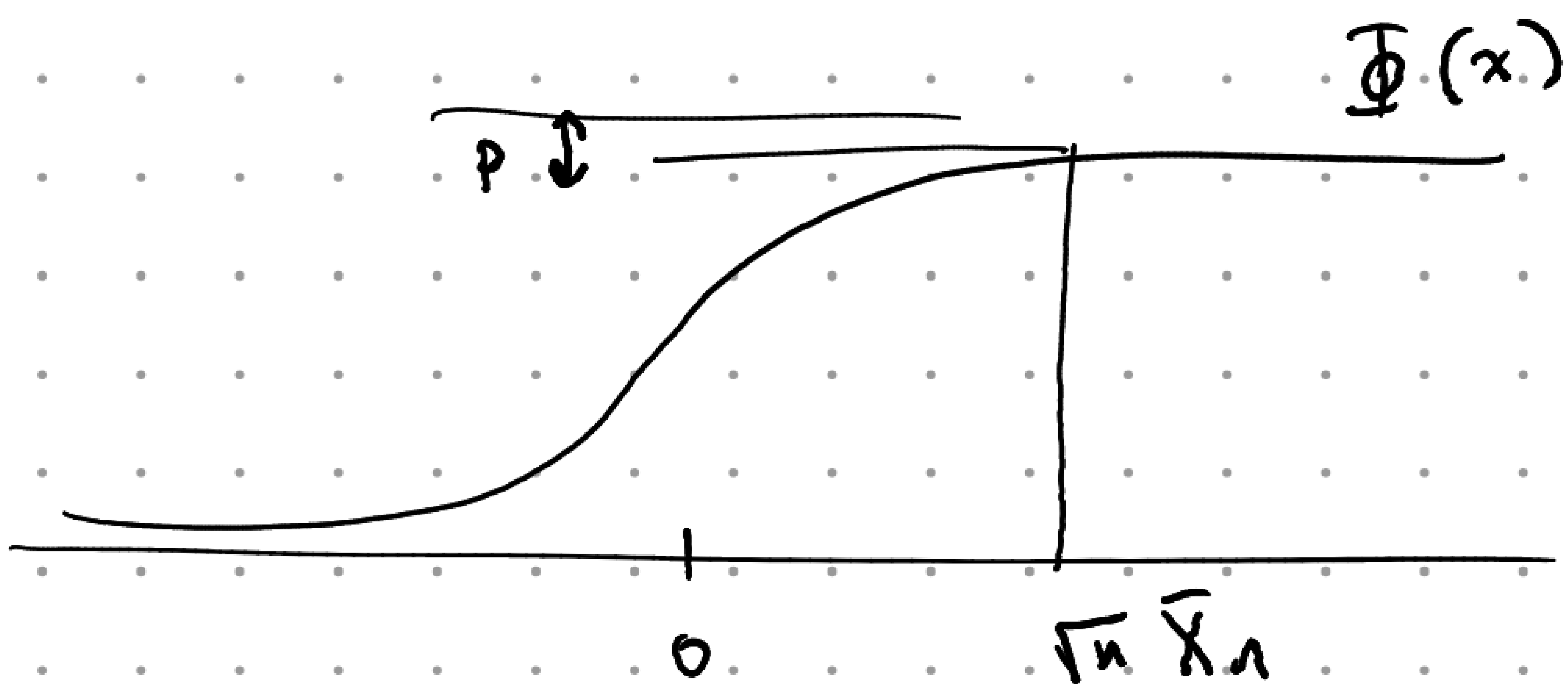
Then p-value $= \inf \{ \alpha : (X_1, \dots, X_n) \in R_\alpha \}$,

Ex. $R_\alpha = \left\{ (x_1, \dots, x_n) : \dfrac{x_1 + \dots + x_n}{n} > \dfrac{\Phi^{-1}(1-\alpha)}{\sqrt{n}} \right\}$

$$p = \inf_\alpha : \frac{X_1 + \dots + X_n}{n} > \frac{\Phi^{-1}(1-\alpha)}{\sqrt{n}}$$

$$= \alpha : \frac{X_1 + \dots + X_n}{n} = \frac{\Phi^{-1}(1-\alpha)}{\sqrt{n}}$$

$$= 1 - \Phi\left(\sqrt{n}(\bar{X}_n)\right)$$



Th. Suppose $R_\alpha = \{ (x_1, \dots, x_n) : T(x_1, \dots, x_n) \geq c_\alpha \}$.

$$\text{p-value} = \sup_{\theta \in \Theta_0} \mathbb{P}\left[T(X_1', \dots, X_n') > T(X_1, \dots, X_n) \,\middle|\, X_1', \dots, X_n' \sim F_\theta \right]$$

p-value is prob of observing more extreme data, given H_0 (i.e. $\theta \in \Theta_0$)