$\mathcal{F} = \{F_\theta : \theta \in \Theta\}$

$H_0 : \{\theta \in \Theta_0\}$

$H_1 : \{\theta \in \Theta_0^c\} \longrightarrow \Theta \setminus \Theta_0$

Get data $X_1, \dots, X_n \sim F_\theta$

Given $R \subseteq \mathbb{R}^n$, reject H_0 if $(X_1, \dots, X_n) \in R$

usually: $R = \{(x_1, \dots, x_n) : T(x_1, \dots, x_n) > c\}$

for some $T : \mathbb{R}^n \to \mathbb{R}$, $c \in \mathbb{R}$

		retain H_0	reject H_0
Truth	$\theta \in \Theta_0$	correct	Type I
	$\theta \in \Theta_0^c$	Type II	correct

$$\beta(\theta) = \mathbb{P}\left[(X_1, \ldots, X_n) \in R \,\middle|\, X_1, \ldots, X_n \sim F_\theta\right]$$

$$\text{size} = \sup_{\theta \in \Theta_0} \beta(\theta)$$

level $-\alpha$ if $\quad$ size $\leq \alpha$

<u>Def.</u> Suppose $\forall \alpha \in (0,1)$ we have a size α test with rejection region R_α

The p-value $= \inf \{ \alpha : (X_1, \ldots, X_n) \in R_\alpha \}$, Ex. $X_1, \dots, X_n \sim N(\mu, 1)$

$$\Theta_0 = \{\mu \le 0\}, \quad \Theta_1 = \{\mu > 0\}$$

Test: reject H_0 if $\bar{X}_n > c$

$$R = \left\{ (x_1, \dots, x_n) : \frac{x_1 + \dots + x_n}{n} > c \right\}$$

Q. what should I do to c to get a really small α-level?

$$\beta(\mu) = \mathbb{P}\left[\bar{X} > c \mid X_1, \dots, X_n \sim N(\mu, 1)\right]$$

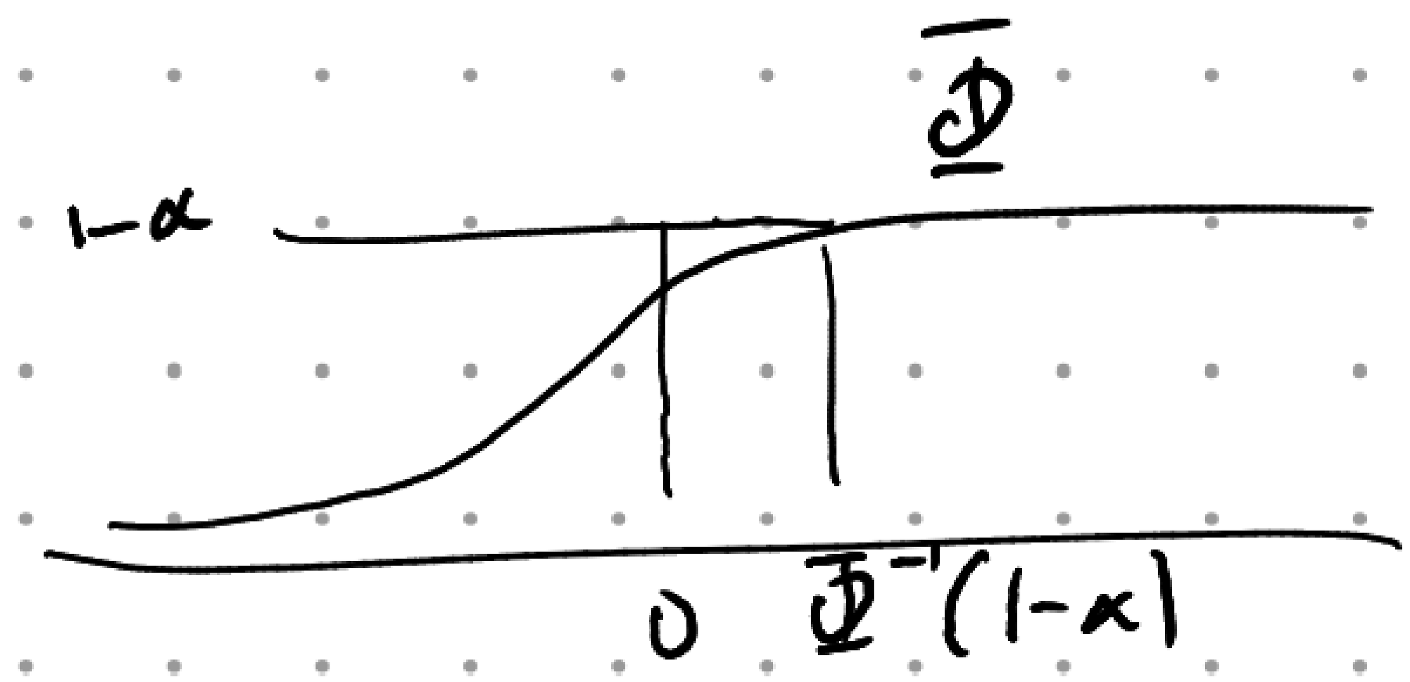
$$= \mathbb{P}\left[\sqrt{n}(\bar{X} - \mu) > \sqrt{n}(c - \mu) \mid X_1, \dots, X_n \sim N(0, 1)\right]$$

$$= \mathbb{P}\left[Z \ge \sqrt{n}(c - \mu) \mid Z \sim N(0,1)\right]$$

$$= 1 - \Phi\left(\sqrt{n}(c - \mu)\right)$$

$$\text{size} = \sup_{\mu \le 0} \beta(\mu) = \beta(0) = 1 - \Phi(\sqrt{n} \cdot c)$$

β inc. in μ



level α if $c \ge \dfrac{\Phi^{-1}(1-\alpha)}{\sqrt{n}}$

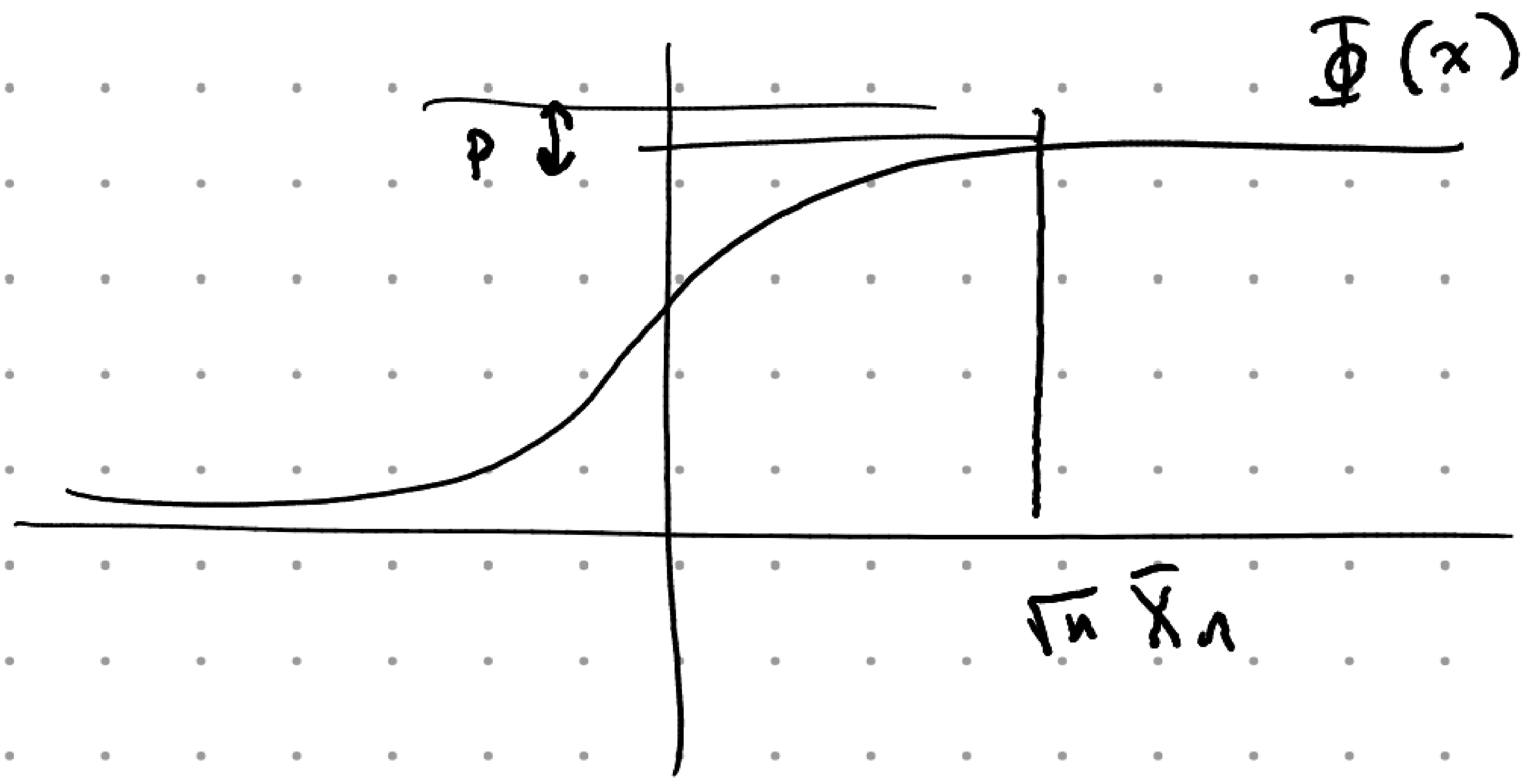
Ex.

$$R_\alpha = \left\{ (x_1, \dots, x_n) : \frac{x_1 + \dots + x_n}{n} > \frac{\Phi^{-1}(1-\alpha)}{\sqrt{n}} \right\}$$

$$p = \inf_\alpha : \frac{X_1 + \dots + X_n}{n} > \frac{\Phi^{-1}(1-\alpha)}{\sqrt{n}}$$

$$= \alpha : \frac{X_1 + \dots + X_n}{n} = \frac{\Phi^{-1}(1-\alpha)}{\sqrt{n}}$$

$$= 1 - \Phi\left(\sqrt{n}(\bar{X}_n)\right)$$



Suppose $\theta \in \Theta_0$ and we sample $X_1', \dots, X_n' \sim F_\theta$.

$$\mathbb{P}\left[T(X_1', \dots, X_n') > T(X_1, \dots, X_n) \mid X_1, \dots, X_n\right] \leq \text{p-value}$$