Hypothesis testing

Ex.

H_0 = message is not spam

H_1 = message is spam

null hypothesis = default assumption

alternate hypothesis = alternate assumption

Ex $\Theta_0 = \{\theta_0\}$

$\Theta_0 = \{\theta : \theta \leq \theta_0\}$

Hypothesis test

Construct statistic $T(X_1, \ldots, X_n)$, and pick rejection region $R = \{(x_1, \ldots, x_n) : T(x_1, \ldots, x_n) > c\}$

- $(X_1, \ldots, X_n) \in R \Rightarrow$ reject H_0
- $(X_1, \ldots, X_n) \notin R \Rightarrow$ retain H_0

		retain H_0	reject H_0
Truth	$\theta \in \Theta_0$	correct	Type I error
	$\theta \in \Theta_0^c$	Type II error	correct

- Assume innocent unless we have strong evidence of guilt
- Assume null, unless strong evidence otherwise.
- "better to let 10 guilty people free than falsely convict 1 innocent person"
- Hypothesis testing cares about incorrectly deviating from status quo
  - philosophical assumption that Type I error bad, Type II error not important

		mark as not spam	mark as spam
Truth	not spam	correct	very bad
	spam	annoying	correct

Def. power function of test w. rejection region R is

$$\beta(\theta) = \mathbb{P}\left[(X_1, \ldots, X_n) \in R \mid X_1, \ldots, X_n \sim F_\theta \right]$$

Ideally: $\beta(\theta) = \begin{cases} 0 & \text{if } \theta \in \Theta_0 \\ 1 & \text{if } \theta \in \Theta_0^c \end{cases}$

Def. The size of a test is

$$\alpha = \sup_{\theta \in \Theta_0} \beta(\theta)$$

Def. A test has level α if size is $\leq \alpha$

Type I error

$$\theta \in \Theta_0 \Rightarrow \mathbb{P}[\text{reject } H_0] = \beta(\theta) \leq \alpha$$

$$\mathbb{P}[\text{retain } H_0] = 1 - \beta(\theta) \geq 1 - \alpha$$

correct

$$\theta \in \Theta_0^c \Rightarrow \mathbb{P}[\text{reject } H_0] = \beta(\theta)$$

no relation to level α

$$\mathbb{P}[\text{retain } H_0] = 1 - \beta(\theta)$$

Type II error