Delta Method

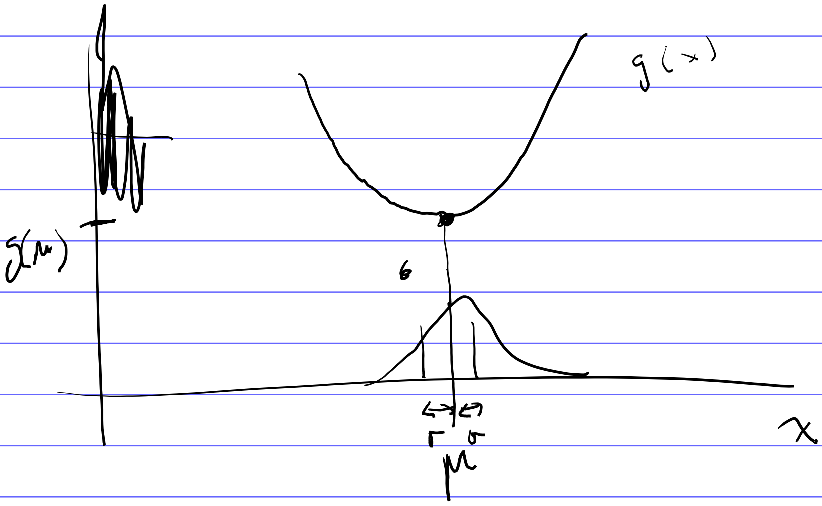
Suppose $\sqrt{n}\dfrac{(Y_n - \mu)}{\sigma} \xrightarrow{d} N(0,1)$

Suppose also g is differentiable with $g'(\mu) \neq 0$. Then

$$\sqrt{n}\,\frac{\left(g(Y_n) - g(\mu)\right)}{|g'(\mu)|\,\sigma} \to N(0,1)$$

$Y_n \approx N(\mu, \frac{\sigma^2}{n})$ then

$$g(Y_n) \approx N\left(g(\mu),\ g'(\mu)\frac{\sigma^2}{n}\right)$$



goal $Y \sim \text{Ber}(p) = \begin{cases} 1 & \text{w.p. } p \\ 0 & \text{w.p. } 1-p \end{cases}$

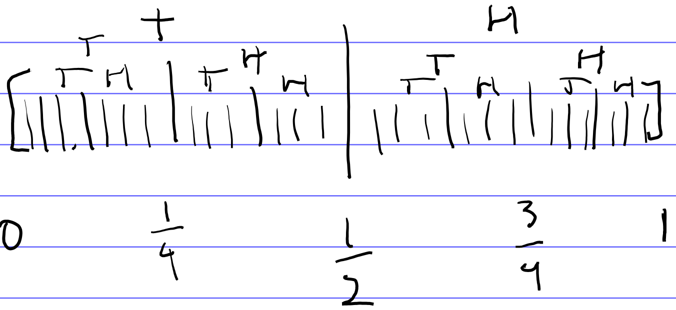
$X \sim \text{Unif}(0,1)$

$$Y = \begin{cases} 1 & \text{if } X > 1-p \\ 0 & \text{if } X < 1-p \end{cases}$$

$$\mathbb{P}[Y=1] = \mathbb{P}[X > 1-p] = \mathbb{P}[X \in (1-p, 1)] = p$$

$$\mathbb{P}[Y=0] = \ldots = 1-p$$

Why is sampling unif(0,1) easy?



Inverse CDF sampling

Goal: sample Y from dist fxn F_Y

Given: $X \sim \text{Unif}(0,1)$

Let $Y = g(X)$ suppose g is non-decreasing

$g^{-1}(y) \in [0,1]$

$$F_Y(y) = \mathbb{P}[Y \le y] = \mathbb{P}[g(X) \le y]$$

$$= \mathbb{P}[X \le g^{-1}(y)]$$

$$= F_X(g^{-1}(y)) \qquad F_X(x) = \begin{cases} x & \text{if } x \in [0,1] \\ 0 & x < 0 \\ 1 & x > 1 \end{cases}$$

$$= g^{-1}(y)$$

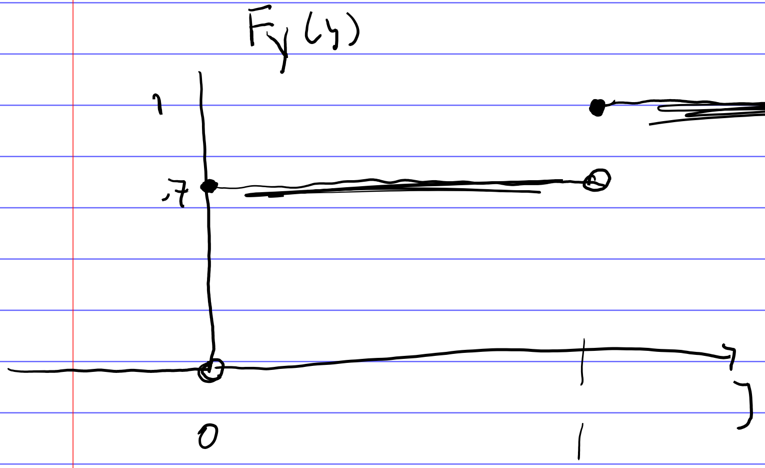
$$F_Y(y) = g^{-1}(y)$$

$$g(x) = F_Y^{-1}(x)$$

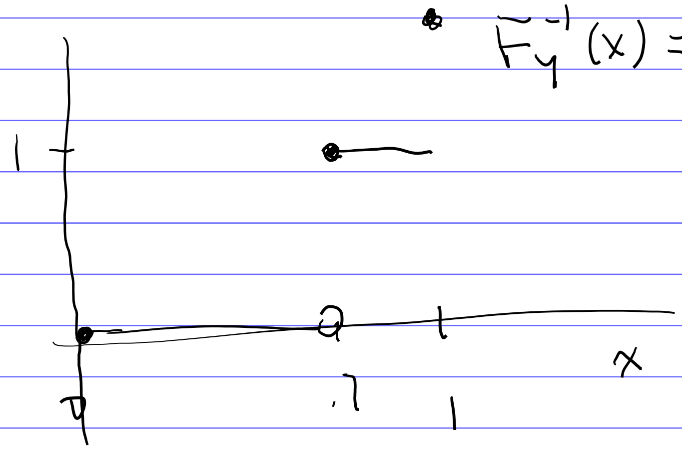
$Y = F_Y^{-1}(X)$ has distribution F_Y

$Y \sim Ber(.3) = \begin{cases} 1 & \text{wp. } .3 \\ 0 & \text{wp. } .7 \end{cases}$

$X \sim Unif(0,1)$ then $Y = \begin{cases} 1 & \text{if } X > .7 \\ 0 & \text{o-w.} \end{cases}$



$F_Y^{-1}(x) = \begin{cases} 1 & \text{if } x \geq .7 \\ 0 & \text{if } < .7 \end{cases}$



$$F_Y(y) = \int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{t^2}{2}\right) dt$$