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courses.chen.pw/mathstats_f2022

Edstem = discussion
- can insert equations
- supports threading
- don't email me non-personal questions

Gradescope = turn in HW / grade HW
- HW must be <u>legible</u>
- tag each question
- no late HW (email me ahead of time)

Bagel Institute = live questions in lecture
- will test it out
- hopefully makes it easier for people to ask questions

Grading

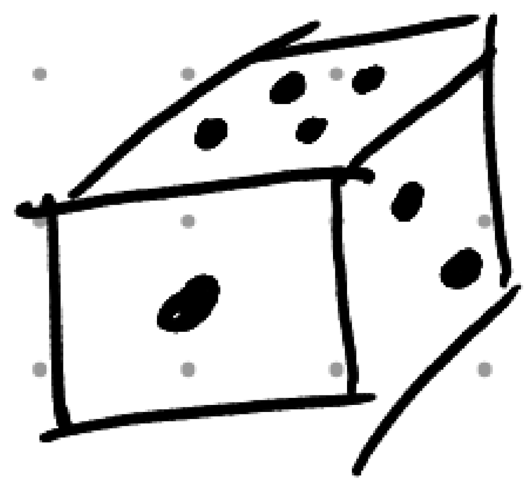
15% participation

30% homework

25% quizzes - meant to test critical concepts. Typically very similar to a past HW problem

30% final project

Probability Review



roll once $\rightarrow \underbrace{\{1,2,3,4,5,6\}}_{\text{sample space}} = \Omega$

"possible outcomes of experiment"

Where does <u>randomness</u> enter?

- if we place dice on given side, this is deterministic
- if we roll dice, this is still deterministic (at least if we know initial conditions)
- We can abstract away the uncertainty in initial conditions...

How likely am I to roll a given side?

- objective / frequentist: roll dice many times, and tally up results.

1	2	3	4	5	6
|	|	|	||	||||	|

- subjective: based on belief
  - e.g. what we might bet based on

Events

An *event* is a collection of possible outcomes

Ex. $\{1,3,5\} = \{\text{roll odd \#}\}$

$\{1\} = \{\text{roll one}\}$

$\{1,2,3,4,5,6\} = \{\text{any face}\}$

Events are subsets of $\underbrace{\text{power set}}_{\text{set of all subsets}}$ of sample space

How likely is an event?

We write $\mathbb{P}[A]$ for the probability of an event A

- What are probability of above events?

$\mathbb{P}[\{1,3,5\}] = \frac{1}{2}$

$\mathbb{P}[\{1\}] = \frac{1}{6}$

$\mathbb{P}[\{1,2,3,4,5,6\}] = 1$

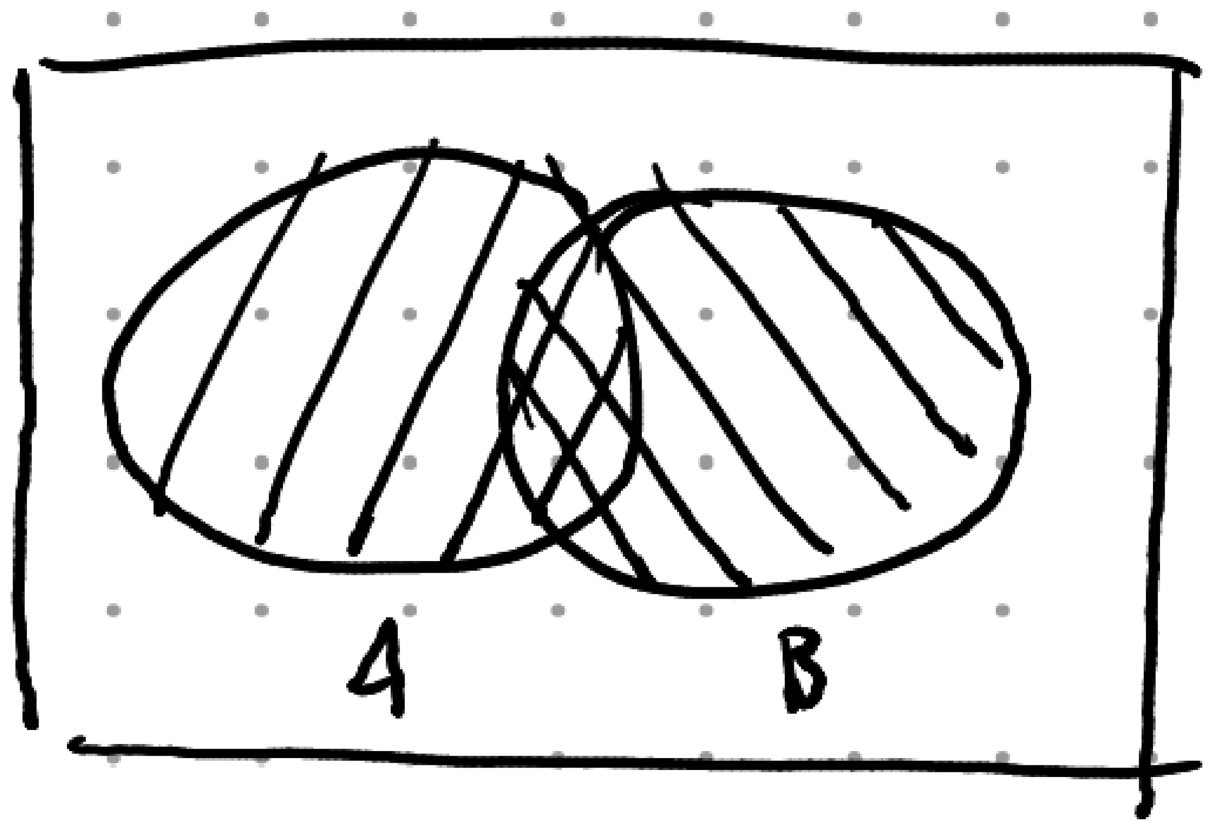
- These probabilities are assigned based on the assumption that dice is fair
- fair: $\mathbb{P}[\{1\}] = \mathbb{P}[\{2\}] = \dots = \mathbb{P}[\{6\}]$
  - but how do we get probs. for other events?

A <u>probability distribution</u> assigns a real #
to each event such that

1. $\mathbb{P}[A] \geq 0 \quad \forall A$
2. $\mathbb{P}[\Omega] = 1$
3. if $A_1, A_2, \ldots$ disjoint, $\mathbb{P}[A_1 + A_2 + \cdots] = \mathbb{P}[A_1] + \mathbb{P}[A_2] + \cdots$

Can derive useful properties

- $\mathbb{P}[\emptyset] = 0$
  - $\mathbb{P}[\Omega + \emptyset] = \mathbb{P}[\Omega] + \mathbb{P}[\emptyset] = 1 + \mathbb{P}[\emptyset]$
  - $\mathbb{P}[\Omega + \emptyset] = \mathbb{P}[\Omega] = 1$
- $A \subseteq B \implies \mathbb{P}[A] \leq \mathbb{P}[B]$
- $\mathbb{P}[A] \in [0, 1]$
- $\mathbb{P}[A^c] = 1 - \mathbb{P}[A]$
- $\mathbb{P}[A \cup B] = \mathbb{P}[A] + \mathbb{P}[B] - \mathbb{P}[A \cap B]$



Independence

Events A and B are *independent* if

$$\mathbb{P}[A \cap B] = \mathbb{P}[A]\,\mathbb{P}[B]$$

- occurrence of one event does not impact prob. of occurrence of other.
- sometimes we will assume independence
- sometimes we will show/prove independence

Ex. Let $A = \{1,3,5\}$ $B = \{3,4,5,6\}$

Are A and B independent?

$A \cap B = \{3,5\}$ $\quad \mathbb{P}[A \cap B] = \frac{1}{3}$

$\mathbb{P}[A] = \frac{1}{2}$ $\quad \mathbb{P}[B] = \frac{2}{3}$ $\quad \Rightarrow \mathbb{P}[A]\mathbb{P}[B] = \frac{1}{3}$

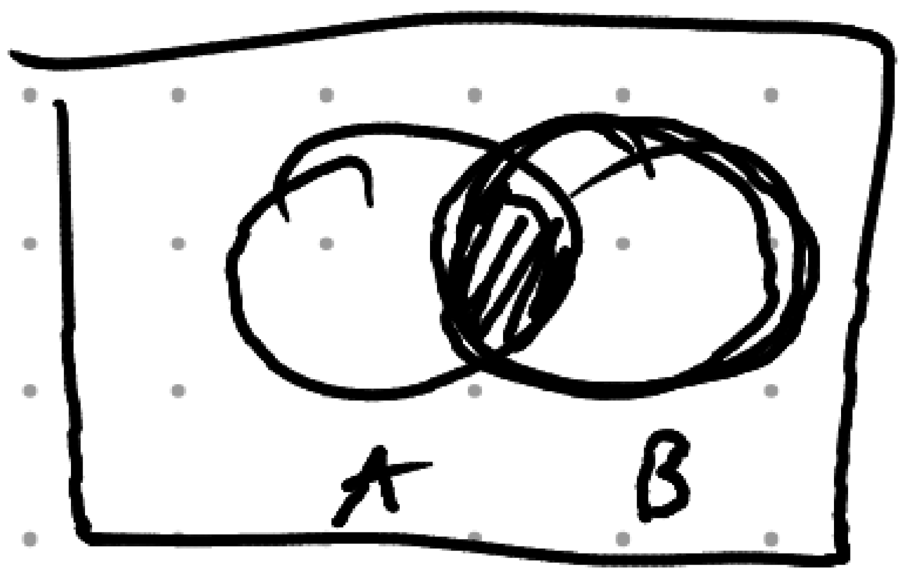
Events are independent!

Conditional Probability

The *conditional probability* of A given B is

$$\mathbb{P}[A|B] = \mathbb{P}[A \cap B] / \mathbb{P}[B]$$

$\mathbb{P}[\{1\} \mid \{odd\}] = \mathbb{P}[1 \text{ and odd}]/\mathbb{P}[odd] = \frac{1/6}{1/2} = \frac{1}{3}$



A, B indepen $\iff \mathbb{P}[A \mid B] = \mathbb{P}[A]$

Law of total probabil

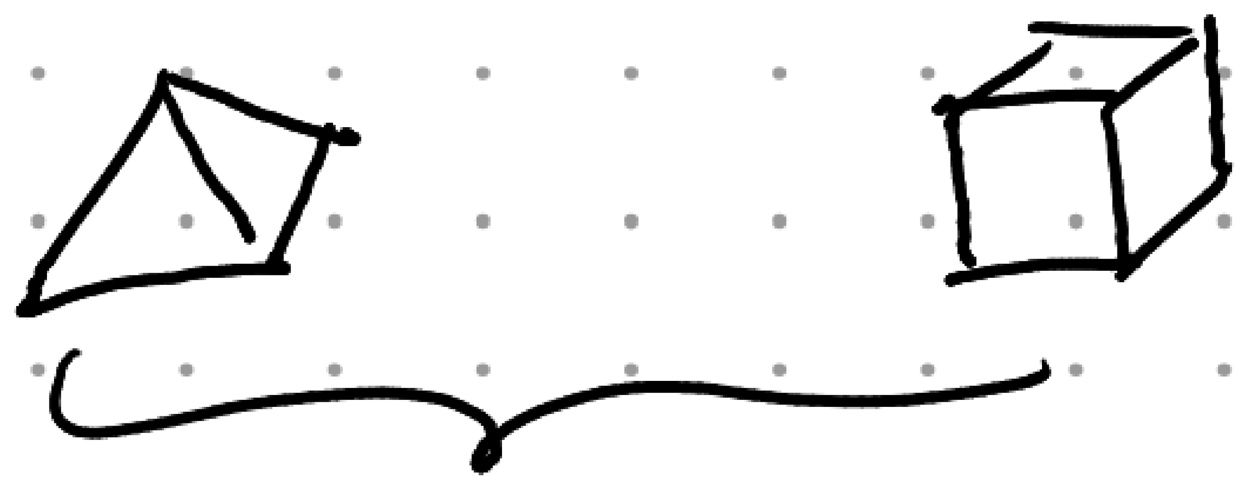
$$\mathbb{P}[B] = \mathbb{P}[B \mid A_1]\,\mathbb{P}[A_1] + \mathbb{P}[B \mid A_2]\,\mathbb{P}[A_2]$$

given $A_1 \cap A_2 = \phi$, $A_1 \cup A_2 = \Omega$

Bayes theorem

$$\mathbb{P}[A \mid B] = \frac{\mathbb{P}[B \mid A]\,\mathbb{P}[A]}{\mathbb{P}[B]}$$

D_4 D_6



flip fair coin H = use D_4
T = use D_6

roll dice 2x

- what is the sample space?

$\{(1,1), (1,2), \dots (2,1) \dots (6,6)\}$

$= \{(a,b) : a, b \in \{1,2,3,4,5,6\}\}$

- what is the probability the second dice has a 6?

- what is the probability the second dice has a 6 given the first one has a 6?
a 1?

Random Variables

A random variable is a map from the sample space to reals

Ex. dice
$\Omega = \{1,2,3,4,5,6\}$
$X(\omega) = \omega \quad \forall \omega \in \Omega$

Ex. flip coin 5x $\quad \Omega = \{(c_1 c_2 c_3 c_4 c_5) : c_i \in \{H,T\}\}$
$X(\omega) = \#$ heads
$X(HTTHH) = 3$

Cumulative distribution function

$F_X(x) = \mathbb{P}[X \leq x]$

Probability mass function
$f_X(x) = \mathbb{P}[X = x] \quad \longleftrightarrow \quad \mathbb{P}[X \in [a,b]] = \int_a^b f_X(x)\,dx$

$F_X(x) = \sum_{x_i \leq x} f_X(x_i) \longleftrightarrow F_X(x) = \int_{-\infty}^{x} f_X(x)\,dx$

If X discrete, then f_X has delta distributions.