Homework 1 — Linear Algbera I

Instructions:

- Due Feb 5 at 11:59pm on Gradescope.
- You must follow the submission policy in the syllabus

Problem 1 (Complex Numbers)**.**

(a) Show that $\alpha + \beta = \beta + \alpha$ for all $\alpha, \beta \in \mathbb{C}$.

(b) Show that $\lambda(\alpha + \beta) = \lambda\alpha + \lambda\beta$ for all $\alpha, \beta, \lambda \in \mathbb{C}$.

Problem 2 (Vector Spaces)**.**

(a) Let V be a vector space. Prove that $-(-\vec{v}) = \vec{v}$ for all $\vec{v} \in V$.

(b) Prove $0\vec{v} = \vec{0}$ for all $\vec{v} \in V$.

(c) Let $V = \mathbb{R}^n$. Suppose $a \in \mathbb{R}$ and $v \in V$. Prove that if $a\vec{v} = \vec{0}$, then $a = 0$ or $\vec{v} = \vec{0}$.

(d) The empty set $\{\}$ is not a vector space. Which property of vector spaces is not satisfied by the empty set?

Problem 3 (Subspaces)**.**

(a) Prove that if X and Y are linear susbpaces of V, then so is $X + Y$.

(b) Prove that the set $\{0\}$ consisting of the zero element of V is a subspace of V.

(c) Suppose $b \in \mathbb{R}$. Show that the set of continuous real-valued functions f on the interval $[0, 1]$ such that $\int_0^1 f(x)\mathrm{d}x = b$ is a subspace of the vector space of all continuous real-valued functions on $[0, 1]$ if and only if $b = 0$.

Problem 4. Let $(V, +, \cdot, \mathbb{F})$ be a vector space. Suppose U is a non-empty subset of V closed under vector addition and scalar multiplication; i.e. for all $a, b \in \mathbb{F}$ and $\vec{u}, \vec{v} \in V$, $a\vec{u} + b\vec{v} \in U$. Prove that $(U, +, \cdot, \mathbb{F})$ is a vector space.